Supplemental Document

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Polarization-artifact reduction and accuracy improvement of Jones-matrix polarization-sensitive optical coherence tomography by multi-focus-averaging based multiple scattering reduction: supplement

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Polarization-artifact reduction and accuracy improvement of Jones-matrix polarization-sensitive optical coherence tomography by multi-focus-averaging based multiple scattering reduction: supplemental document

This file supplements Section 4.2.1 by showing the proof of a formula used in the section. This file also supplements Section 4.5 and Fig. 8 by showing the photographs, degree-of-polarization uniformity (DOPU) images, and birefringence images of two additional fishes (Samples 2 and 3) measured for validation of MS reduction by the multi-focus averaging (MFA).

1. PROOF OF FORMULA

We have used the following formula in Section 4.2.1.

$$(\mathbf{A}+\mathbf{B})^{-1} = \mathbf{A}^{-1} - (\mathbf{I}+\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{-1}, \tag{S1}$$

where $\mathbf{A}$ and $\mathbf{B}$ are arbitrary nonsingular square matrices. This formula can be proven as follows.

We hypothesize that the formula [Eq. (S1)] is correct, and then show that this hypothesis leads us to an evidently correct equation.

By multiplying $(\mathbf{A}+\mathbf{B})$ from the right, the equation becomes

$$\mathbf{I} = \mathbf{I} + \mathbf{A}^{-1}\mathbf{B} - (\mathbf{I}+\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{A}^{-1}\mathbf{B}\left(\mathbf{I}+\mathbf{A}^{-1}\mathbf{B}\right) \tag{S2}$$

$$\mathbf{A}^{-1}\mathbf{B} = (\mathbf{I}+\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{A}^{-1}\mathbf{B}\left(\mathbf{I}+\mathbf{A}^{-1}\mathbf{B}\right). \tag{S3}$$

By multiplying $(\mathbf{I}+\mathbf{A}^{-1}\mathbf{B})$ from the left, the equation further reformed into

$$(\mathbf{I}+\mathbf{A}^{-1}\mathbf{B})\mathbf{A}^{-1}\mathbf{B} = \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{I}+\mathbf{A}^{-1}\mathbf{B}\right) \tag{S4}$$

$$\mathbf{A}^{-1}\mathbf{B} + \mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{-1}\mathbf{B} = \mathbf{A}^{-1}\mathbf{B} + \mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{-1}\mathbf{B}. \tag{S5}$$

The left- and right-hand sides of the last equation [Eq. (S5)] are evidently identical. Namely, an evidently correct equation was derived from the formula [Eq. (S1)]. So, this proves that the formula is true.

More generalized discussion of this formula can be found in [1].

REFERENCES

1. K. S. Miller, "On the inverse of the sum of matrices," Math. Mag. **54**, 67–72 (1981).

2. SUPPLEMENTARY FIGURE

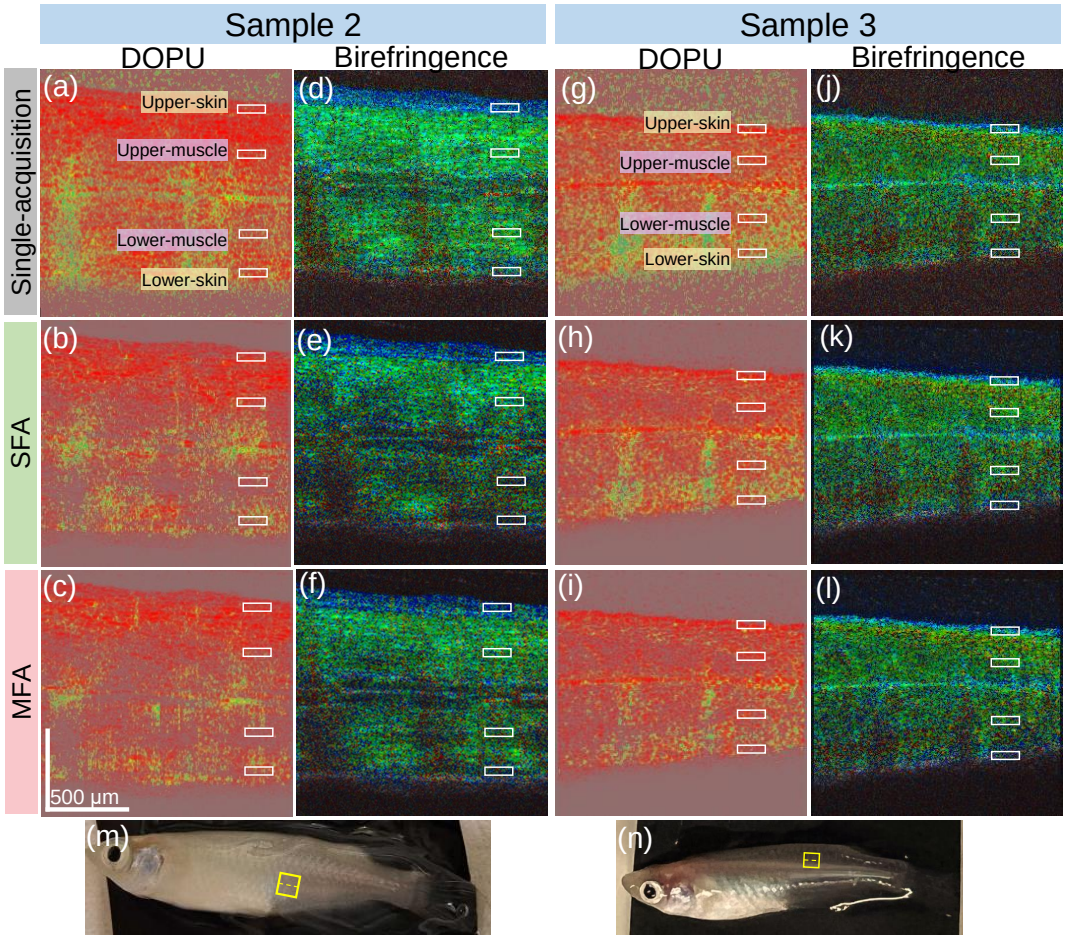

Fig. S1. (a)-(l) are the cross-sectional DOPU and birefringence images of the two additional samples. samples 2 and 3 were two adult medaka fishes. White boxes in (a) and (g) denote the selected ROIs for the skin and muscle regions in the upper- and lower-halves of the samples, respectively. (m) and (n) show the photographs of sample 2 and 3, respectively, where the yellow boxes denote the measured field-of-views, and the yellow dashed lines denote the locations where the cross-sectional images were taken.